nag_prob_normal (g01eac)

1. Purpose

nag_prob_normal (g01eac) returns a one or two tail probability for the standard distribution.

2. Specification

```
#include <nag.h>
#include <nagg01.h>

double nag_prob_normal(Nag_TailProbability tail, double x, NagError *fail)
```

3. Description

The lower tail probability for the standard Normal distribution, $P(X \leq x)$ is defined by:

$$P(X \leq x) = \int_{-\infty}^{x} Z(X)dX$$

where

$$Z(X) = \frac{1}{\sqrt{2\pi}} e^{-X^2/2}, \quad -\infty < X < \infty.$$

The relationship

$$P(X \leq x) = \frac{1}{2} \operatorname{erfc}\left(-\frac{x}{\sqrt{2}}\right)$$

is used, where erfc is the complementary error function, and is computed using nag_erfc (s15adc). For the upper tail probability the relationship $P(X \geq x) = P(X \leq -x)$ is used and for the two tail significance level probability twice the probability obtained from the absolute value of x is returned. When the two tail confidence probability is required the relationship

$$P(X \leq |x|) - P(X \leq -|x|) = \operatorname{erf}\left(\frac{|x|}{\sqrt{2}}\right),$$

is used, where erf is the error function, and is computed using nag_erf (s15aec).

4. Parameters

tail

Input: indicates whether the upper or lower tail probability is required.
If **tail** = **Nag_LowerTail**, the lower tail probability is returned, i.e., $P(X \leq x)$.
If **tail** = **Nag_UpperTail**, the upper tail probability is returned, i.e., $P(X \geq x)$.
If **tail** = **Nag_TwoTailSignif**, the two tail (significance level) probability is returned, i.e., $P(X \geq |x|) + P(X \leq -|x|)$.
If **tail** = **Nag_TwoTailConfid**, the two tail (confidence interval) probability is returned, i.e., $P(X \leq |x|) - P(X \leq -|x|)$.
Constraint: **tail** = **Nag_UpperTail**, **Nag_LowerTail**, **Nag_TwoTailSignif** or **Nag_TwoTailConfid**.

x

Input: the value of the standard Normal variate, x.

fail

The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

If **fail.code** $\neq$ **NE_NOERROR**, then nag_prob_normal returns 0.0.

NE_BAD_PARAM

On entry, parameter **tail** had an illegal value.

6. Further Comments

6.1. Accuracy

Accuracy is limited by ***machine precision***. For detailed error analysis see nag_erfc (s15adc) and nag_erf (s15aec).

6.2. References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* Ch. 7.1, p.297 and Ch. 26.2, p. 931 Dover Publications, New York.

Hastings N A J and Peacock J B (1975) *Statistical Distributions* Butterworth.

7. See Also

nag_erfc (s15adc)
nag_erf (s15aec)

8. Example

Four values of **tail** and **x** are input and the probabilities calculated and printed.

8.1. Program Text

```
/* nag_prob_normal(g01eac) Example Program.
 *
 * Copyright 1996 Numerical Algorithms Group.
 *
 * Mark 4, 1996.
 *
 */


#include <nag.h>
#include <nag_stdlib.h>
#include <stdio.h>
#include <nagg01.h>

main()
{

  double prob;
  double x;

  Integer i;

  Nag_TailProbability tail;
  char tail_char;

  Vprintf(" g01eac Example Program Results\n");

  /*     Skip heading in data file */
  Vscanf("%*[^\n] ");

  Vprintf("\n  Tail    X      Probability \n\n");
  for (i = 1; i <= 4; ++i)
    {
      Vscanf("%c %lf ", &tail_char, &x);

      switch (tail_char)
        {
        case 'L':
          tail=Nag_LowerTail;
          break;
        case 'U':
          tail=Nag_UpperTail;
          break;
        case 'C':
          tail=Nag_TwoTailConfid;
          break;
        case 'S':
          tail=Nag_TwoTailSignif;
        }

      prob = g01eac(tail, x, NAGERR_DEFAULT);

      Vprintf("   %c    %4.2f       %6.4f\n", tail_char, x, prob);
    }
  exit(EXIT_SUCCESS);
}
```

8.2. Program Data

```
g01eac Example Program Data
L  1.96
U  1.96
C  1.96
S  1.96
```

8.3. Program Results

```
 g01eac Example Program Results

  Tail    X      Probability

   L    1.96       0.9750
   U    1.96       0.0250
   C    1.96       0.9500
   S    1.96       0.0500
```